STRUCTURE OF THE NUCLEON: SPIN OBSERVABLES

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I discuss the spin structure of the nucleon at low photon virtualities in the framework of a Lorentz–invariant formulation of baryon chiral perturbation theory. The structure functions of doubly virtual Compton scattering are calculated to one–loop accuracy. The role of the delta and other resonances is analyzed and first steps towards a covariant effective field theory with spin-3/2 fields are outlined. As an example, the quark mass expansion of the delta mass is discussed.

1. Introduction

Understanding the spin structure of the nucleon is a central topic of present nuclear and particle physics activities, for a review see [1]. Of particular interest are certain sum rules which connect information at all energy scales, like e.g. the Gerasimov–Drell–Hearn (GDH) sum rule and its generalization to finite photon virtuality or the Burkhardt–Cottingham (BC) sum rule. Such sum rules are interesting from the theoretical point of view because they constitute moments of the sought after nucleon spin structure functions g_1 and g_2. On the experimental side challenging new meson production experiments using real or virtual photons play an important role since only recently it has become possible to work with polarized beams and polarized targets, thus offering the possibility of mapping out the nucleons' spin structure encoded in these two functions, which can be formulated on a purely partonic (high energy regime) or hadronic level (low energy regime). In both these extreme cases, systematic and controlled theoretical calculations can be performed. The region of intermediate momentum transfer is accessible using quark/resonance models or can be investigated using dispersion relations. In fact, one of the final goals of many of these investigations is to obtain an understanding of how in QCD this transition from the non–perturbative to the perturbative regime takes place, guided by the precise experimental mapping of spin–dependent observables from low momentum transfer to the multi–GeV region, as it is one of the main thrusts of the research carried out e.g. at Jefferson Laboratory.

In this talk, I focus on a theoretical investigation of the nucleon's spin structure in the non-perturbative regime of QCD, utilizing chiral perturbation theory (CHPT) to analyze the structure of the nucleon at low energies. CHPT is based on the spontaneous and explicit chiral symmetry breaking QCD is supposed to undergo (for a general review, see e.g. [2]). By now it is well established that the pion cloud plays an important role in understanding the nucleon's properties in the non–perturbative regime of QCD, and many processes have been analyzed using chiral perturbation theory. Some recent work in various versions/extensions of baryon CHPT pertinent to the topics discussed here can be found e.g. in Refs. [3,4,5,6,7,8].

2. Doubly virtual Compton scattering - formalism

Consider spin–dependent doubly virtual Compton scattering (V^2CS) off nucleons (neutrons or protons) in forward direction, that is the reaction $\gamma^\star(q,\epsilon) + N(p,s) \to \gamma^\star(q,\epsilon') + N(p,s')$, with q (p) the virtual photon (nucleon) four–momentum, s (s') the nucleon spin (polarization) and ϵ (ϵ') the polarization four–vector of the incoming (outgoing) photon. It is common to express the spin amplitude of V^2CS, $T^{[\mu\nu]}(p,q,s)$, in terms of two structure functions, called $S_1(\nu,Q^2)$ and $S_2(\nu,Q^2)$, via

$$T^{[\mu\nu]} = -\frac{i}{2}\,\epsilon^{\mu\nu\alpha\beta}q_\alpha \left\{ s_\beta\, S_1(\nu,Q^2) + [p\cdot q\, s_\beta - s\cdot q\, p_\beta]\,\frac{S_2(\nu,Q^2)}{m^2} \right\}, \quad (1)$$

where s^μ denotes a spin-polarization four-vector, m is the nucleon mass, $\epsilon^{\mu\nu\alpha\beta}$ the totally antisymmetric Levi–Civita tensor, $\nu = p\cdot q/m$ the energy transfer and $Q^2 = -q^2 \geq 0$ the (negative of the) photon virtuality. Note that while $S_1(\nu,Q^2)$ is even under crossing $\nu \leftrightarrow -\nu$, the structure function $S_2(\nu,Q^2)$ is odd. The Compton amplitudes $S_{1,2}(\nu,Q^2)$ are amenable to a chiral expansion. We remark that in what follows, we will mostly be concerned with the reduced amplitudes

$$\bar{S}_i(\nu,Q^2) = S_i(\nu,Q^2) - S_i^{\rm el}(\nu,Q^2)\ , \quad (2)$$

i.e. the Compton amplitudes with the contribution from the elastic intermediate state subtracted. More precisely, these are the contributions from the single nucleon exchange (pole) terms with the corresponding vertices given in terms of the electromagnetic form factors. Only the non-pole parts of the corresponding diagrams contribute to the nucleon spin structure as discussed in more detail below. The relation of V^{2}CS to inelastic electroproduction allows to derive sum rules and moments thereof, like the GDH sum rule, its generalization to finite photon virtuality (which is not unique) or the BC sum rule. For a general discussion of such sum rules and related moments, see e.g. [9]. All these sum rules and their moments can be written in terms of $S_1(\nu, Q^2)$, $S_2(\nu, Q^2)$ using the following dispersion relations [5,6]

$$\begin{aligned} S_1(\nu, Q^2) &= 4e^2 \int_{Q^2/2m}^{\infty} \frac{d\nu' \nu' G_1(\nu', Q^2)}{\nu'^2 - \nu^2} \ , \\ \nu S_2(\nu, Q^2) &= 4e^2 \int_{Q^2/2m}^{\infty} \frac{d\nu' \nu^2 G_2(\nu', Q^2)}{\nu'^2 - \nu^2} \ . \end{aligned} \tag{3}$$

where use has been made of the optical theorem,

$$\text{Im } S_i(\nu, Q^2) = 2\pi \ G_i(\nu, Q^2) \ , \quad (i = 1, 2) \ . \tag{4}$$

Here, $G_1 = g_1/(m\nu)$ and $G_2 = g_2/\nu^2$ are the standard spin-dependent structure functions of deep inelastic scattering. Expanding the structure functions at low energies ν, that is around $\nu = 0$, one obtains the desired set of sum rules. One example is

$$\bar{S}_1^{(0)}(0, Q^2) = 4e^2 \int_{\nu_0}^{\infty} \frac{d\nu' G_1(\nu', Q^2)}{\nu'} = \frac{4e^2}{m^2} I_1(Q^2) \ . \tag{5}$$

Note that the often used first moment $\Gamma_1(Q^2)$ is related to $I_1(Q^2)$ via

$$\Gamma_1(Q^2) = \frac{Q^2}{2m^2} \, I_1(Q^2) \ . \tag{6}$$

3. Chiral expansion of the structure functions

Our calculations are based on an effective chiral pion–nucleon Lagrangian in the presence of external sources (like e.g. photons) supplemented by a power counting in terms of quark (meson) masses and small external momenta. Its generic form consists of a string of terms with increasing chiral dimension,

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \ldots \ . \tag{7}$$

The superscript denotes the power in the genuine small parameter q (denoting pion masses and/or external momenta). A complete one–loop (fourth order) calculation must include all tree level graphs with insertions from all terms given in Eq. (7) and loop graphs with at most one insertion from $\mathcal{L}_{\pi N}^{(2)}$. The complete Lagrangian to this order is given in [12]. For various

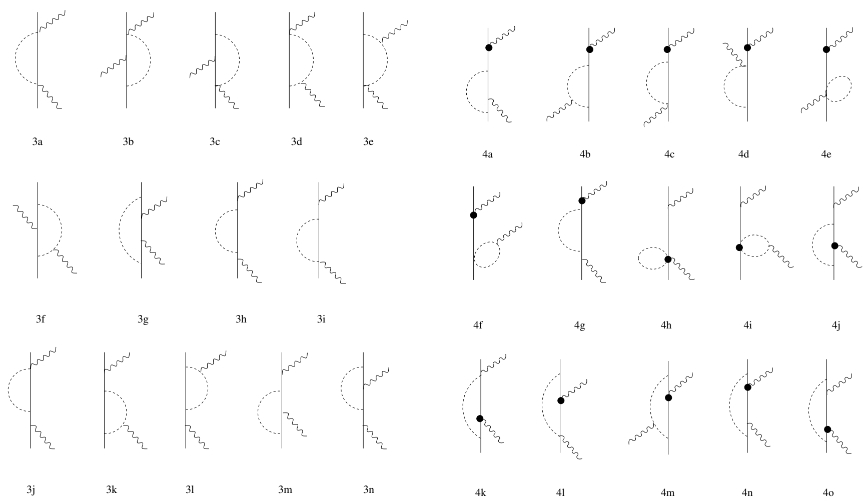

Figure 1. Left: Third order diagrams in Lorentz-invariant baryon CHPT. Soild, dashed and wiggly lines denote nucleons, pions and photons, in order. Right: Fourth order diagrams with exactly one anomalous magnetic moment insertion (filled circle).

reasons (convergence of the p/m expansion in the spin sector, etc, see [10]) we utilize a Lorentz-invariant formulation of baryon CHPT as formulated in[13]. We note that for the case under consideration the only appearing dimension two low–energy constants (LECs), called c_6 and c_7 [2], can be fixed from the anomalous magnetic moment of the proton and of the neutron. Note that there are no contributions from $\mathcal{L}_{\pi N}^{(3,4)}$ for the observables considered here. It is important to work out the complete one-loop amplitudes consisting of third and fourth order contributions since the numerically large values of the LECs $c_{6,7}$ enhance the fourth order terms considerably. Within this approach, we have calculated the reduced structure functions $\bar{S}_{1,2}^{(p,n)}(0, Q^2)$, generically called $\mathcal{S}$. The chiral expansion of $\mathcal{S}$ takes the form

$$\bar{\mathcal{S}} = \bar{\mathcal{S}}^{\rm tree} + \bar{\mathcal{S}}^{\rm loop} \ . \tag{8}$$

The corresponding third and fourth order one-loop diagrams are shown in Fig. 1. In our case, the tree level contribution stems from the remainder of the Born graphs which lead to the following amplitudes

$$T^{\rm Born} = \frac{C(Q^2)}{s - m^2} + (s \to u) + \mathcal{R} \ , \tag{9}$$

with $s = (p+q)^2$, $u = (p-q)^2$, $C(Q^2)$ can be expressed in terms of the nucleon electromagnetic form factors and $\mathcal{R}$ denotes the non–pole (polynomial) remainder from the Born diagrams. Only this latter contribution survives the subtraction of the contribution from the elastic intermediate state.

4. Modeling resonance contributions

It is well-known that the excitation of the $\Delta(1232)$ plays a significant role in the spin sector of the nucleon. One therefore would like to include the delta as a dynamical degree of freedom in the effective Lagrangian. An effective field theory formulation for the relativistic pion–nucleon–delta system is only emerging, as discussed in Sect. 6. Therefore, to get an estimate of the contribution of the Δ-resonance to the various spin structure functions, in [11] we calculated relativistic Born graphs. These are obtained using the standard relativistic spin-3/2 propagator and the $\Delta \to N\gamma$ transition operator. The latter depends on two off-shell parameters X, Y and two transition strengths g_1 and g_2, quantities which are not so well known. We stress that in an effective field theory approach such a dependence on off–shell parameters would be lumped into higher order operators. Bounds on X, Y, g_1 and g_2 have been given in Ref. [14]: $-0.8 \leq X, Y \leq 0.4, 4 \leq g_1 \leq 5$ and $4.5 \leq g_2 \leq 9.5$. Here we constrain g_1 to its large N_c relation, $g_1 = 3(1 + \kappa_p - \kappa_n)/2\sqrt{2} = 5.0$ and use two sets of parameters, $X = Y = 0.4$, $g_2 = 4.5$, and $X = Y = -0.8$, $g_2 = 9.5$, respectively. We note that these bounds are very conservative, a more precise determination based on a combined reanalysis of spin–independent Compton scattering and pion electroproduction based on covariant baryon CHPT would certainly lead to more stringent bounds. Of course, there are also smaller contributions from higher baryon resonances, but we do not include them in this work.

A less pronounced though important resonance contribution is related to the vector mesons. Again, a systematic EFT prescription how to include these degrees of freedom is only just emerging [15,16]. We adopt here the procedure advocated in Ref.[17]. In the pion–nucleon EFT, any vector meson contribution is hidden in the values of the various LECs. However, the momentum dependence of the vector meson propagator is only build up slowly by adding terms of ever increasing chiral dimension. This can be done much more efficiently by including vector mesons in a chirally symmetric manner and retaining the corresponding dimension two counterterms, so that the

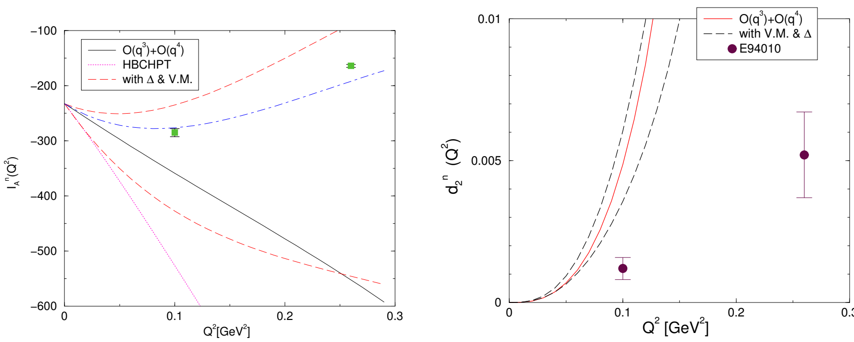

Figure 2. Left: The integral $I_A(Q^2)$ (see Ref. [9]) for the neutron in units of μb. The solid line gives the fourth order result, the dashed lines represent the theoretical uncertainty due to variation in the delta parameters. Dot-dashed line: delta parameters are adjusted to reproduce the data. For comparison, the HBCHPT result is depicted by the dotted line. The data are from Ref. [22]. Right: The second moment d_2^n in comparison to the data of Ref.[23].

LECs c_6, c_7 are effectively replaced by [17]

$$c_6 \longrightarrow \hat{c}_6 + g_{\rho NN}\,\kappa_\rho\,\frac{F_\rho M_\rho}{M_\rho^2 - t}\ , \tag{10}$$

$$c_7 \longrightarrow \hat{c}_7 - \frac{g_{\rho NN}\,\kappa_\rho}{2}\,\frac{F_\rho M_\rho}{M_\rho^2 - t} + \frac{g_{\omega NN}\,\kappa_\omega}{2}\,\frac{F_\omega M_\omega}{M_\omega^2 - t} + \frac{g_{\phi NN}\,\kappa_\phi}{2}\,\frac{F_\phi M_\phi}{M_\phi^2 - t}\ .$$

Here, t is the invariant four–momentum squared and the remainders $\hat{c}_6, \hat{c}_7$ account for physics not related to vector mesons. They have been determined from fitting the nucleons electromagnetic radii [17]. All other parameters appearing in Eqs.(11) can be taken form the dispersion–theoretical analysis of Refs. [18].

5. Results and discussion

Here I discuss a few selected results, for more details the reader is referred to the papers [10,11] and the talks by S. Choi [19], A. Deur [20] and G. Dodge [21] given at this conference. The chiral expansion of the structure functions and their moments is discussed in [10], in particular also the comparison to the heavy baryon results obtained e.g. in Refs.[5,6]. The inclusion of resonance contributions as described in the preceding section allows for a better comparison with the data, as discussed in detail in Ref.[11]. Two typical results are shown in Fig. 2. In the left panel, the prediction of the integral $I_A(Q^2)$ (as defined by Drechsel et al. [9]) for the neutron is given in comparison to the data from JLab [22]. It is evident that for photon virtualities above 0.15 GeV^2, a pure chiral description at one-loop is insufficient. In the right panel, the recently measured moment d_2 for the neutron is shown,

$$d_2(Q^2) = \int_0^1 dx\, x^2\, \left[g_2(x,Q^2) - g_2^{WW}(x,Q^2)\right]\ , \tag{11}$$

which measures higher twist contributions to the spin structure function g_2 (i.e. the deviation from the Wandzura-Wilczek relation). Interestingly, these data can not be described even when the delta and vector mesons are included already at very small photon virtualities (such a result is also found in the heavy baryon scheme, see Choi's talk and Ref.[25]).

6. Lorentz-invariant baryon CHPT with spin-2/3 fields

To overcome the model-dependent calculation of the delta contribution to the various sum rules and moments, one must extend the covariant effective field theory method to spin-3/2 fields. As in the corresponding heavy fermion scheme, the so-called "small scale expansion" of Ref.[26], one treats the nucleon-delta splitting $\Delta \equiv m_\Delta - m_N$ as an additional small parameters (the others being external momenta and the pion mass). Every observable can then be expanded in the small scale ε, where ε collects all small parameters. However, one has to assure that the so formulated theory fulfills decoupling (see e.g. the discussion in Ref.[27]). Furthermore, since Δ stays finite in the chiral limit of vanishing up and down quark masses, this is a phenomenological extension of chiral QCD, but based on a consistent power counting. In Ref.[24], a Lorentz–invariant formulation of baryon chiral perturbation theory including spin-3/2 fields was presented. Particular attention has to be paid to the projection on the spin-3/2 components of the delta fields. To make this point more clear, consider the standard delta propagator in d space-time dimensions,

$$\mathcal{G}^{\Delta}_{\mu\nu}(p) = -\frac{\not{p} + m_\Delta}{p^2 - m_\Delta^2}\, P^{3/2}_{\mu\nu} + \text{spin} - 1/2 \ \ \text{components}\,, \tag{12}$$

$$P^{3/2}_{\mu\nu} = g_{\mu\nu} - \frac{1}{d-1}\,\gamma_\mu\gamma_\nu - \frac{1}{(d-1)\,p^2}\left(\not{p}\gamma_\mu p_\nu + p_\mu\gamma_\nu\not{p}\right) - \frac{d-4}{d-1}\,\frac{p_\mu p_\nu}{p^2}\ .$$

Note the infrared singular pieces $\sim 1/p^2$ appearing in the spin-projected parts of the propagator, which require a special treatment. In fact, in Ref.[24] the prescription of Becher and Leutwyler to generate the IR singular part from a one-loop integral was extended to deal with such new structures that do not appear in the pion–nucleon EFT. For details, I refer to Ref.[24]. Also, we remark that the spin-1/2 pieces do not propagate and thus one is able to absorb their contribution in purely polynomial terms (which amounts to a redefinition of certain low-energy constants in the effective field theory). This was shown for the explicit case of the nucleon mass in Ref.[24].

As a definite example, consider the quark mass expansion of the nucleon and the delta mass to third order in ε,

$$\begin{aligned} m_N &= m_0 - 4c_1 M_\pi^2 + g_A^2 J_a \ , \\ m_\Delta &= m_0^\Delta - 4c_1^\Delta M_\pi^2 + c_A^2 J_b + h_A^2 J_c \ , \end{aligned} \tag{13}$$

where the loop integral J_i corresponds to the diagram i ($i = a, b, c$) in Fig. 3. Here, g_A, c_a and h_A are the leading nucleon, $N\Delta$ and delta axial

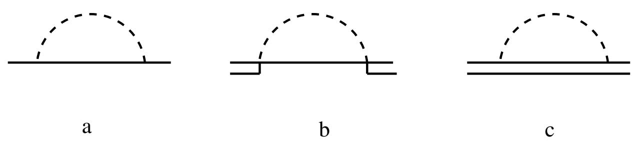

Figure 3. Lowest order self-energy diagrams. Solid/double/dahed lines denote nucleons, deltas and pions, in order.

coupling constants, we use $g_A = 1.267$, $c_A = 1.575$ from the imaginary part of the complex delta-pole and $h_A = 9g_A/5$ from SU(6). c_1 and c_1^Δ are the LECs related to the leading explicit chiral symmetry-breaking terms in the effective Lagrangian. We have $c_1 \simeq -1\,\mathrm{GeV}^{-1}$ from various analyses of pion-nucleon scattering in CHPT [28] and $c_1^\Delta = c_1$ if one assumes SU(6) for simplicity. Furthermore, $m_0^\Delta = m_0 + \Delta_0$, with $m_0 = 0.88\,\mathrm{GeV}$ [29] and $\Delta_0 = 0.271\,\mathrm{GeV}$ from the complex delta-pole. In Fig. 4 the (preliminary) results on the pion mass expansion of m_N and m_Δ to $\mathcal{O}(\varepsilon^3)$ are shown. The bands are obtained by varying the scale of dimensional regularization in a fairly large interval (note that in IR baryon CHPT it is natural to set this scale equal to the mass of the heavy fermion). The third order results are clearly only useful for pion masses below 400 MeV, a fourth order calculation is needed to extend this range (see also the discussion in [29]). A more detailed account of this work with many other applications and a comparison to the early work of Ref.[30] on the quark mass expansion of the N and delta masses will be given in Ref.[31].

7. Summary and outlook

This talk was concerned with the investigation of sum rules and their moments obtained from doubly virtual Compton scattering at low photon vir-

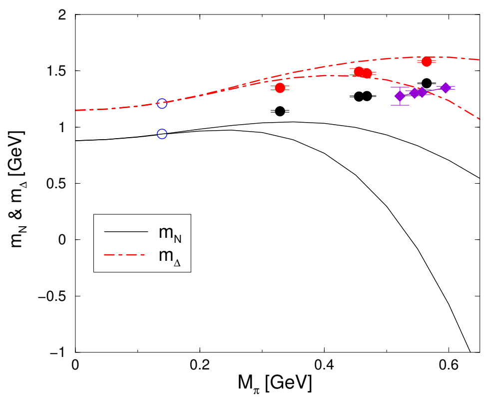

Figure 4. Pion mass expansion of the nucleon (solid lines) and the delta mass (dot-dashed lines) compared to lattice results from UKQCD and CP-PACS.

tualities, which is studied experimentally in great detail at Jefferson Lab. The calculations presented are based on a Lorentz-invariant formulation of baryon CHPT. The spin structure functions at fourth order are given free of unknown LECs. In addition, the resonance contribution from the delta and from vector mesons as described in sect. 4. As a new development, a Lorentz-invariant formulation for spin-3/2 fields was discussed, which should ultimately be applied to the various observables obtained from $\mathrm{V^2CS}$. Also, precise data at very low photon virtualities ($Q^2 \le 0.1\,\mathrm{GeV}^2$) are required to obtained more stringent tests of the chiral structure of QCD.

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